Current drive generation based on autoresonance and intermittent trapping mechanisms

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(Received 11 March 1999)

Two mechanisms for generating streams of high-velocity electrons are presented. One has its origin in auto resonance (AR) interaction, which takes place in the system after a trapping conditioning stage, the second being dominated by the trapping process itself. These mechanisms are revealed from the study of the relativistic motion of an electron in a configuration consisting of two counterpropagating electromagnetic waves along a constant magnetic field in a dispersive medium. Using a Hamiltonian formalism, we have numerically solved the equations of motion and presented the results in a set of figures showing the generation of streams of electrons having high parallel velocities. Insight into these numerical results is gained from a theoretical analysis, which consists of a reformulation of the equations of motion. The operation of these mechanisms was found to circumvent the deterioration of the electron acceleration process that is characteristic for a dispersive medium, thus allowing for an effective generation of a current drive. Discussion of the results follows. [S1063-651X(99)08809-1]

PACS number(s): 52.35.Nx, 52.20.Dq, 05.45.−a, 52.50.Gj

I. INTRODUCTION

The possibility to utilize electromagnetic waves in the electron cyclotron range of frequencies for heating plasmas is successfully realized in many laboratories around the world. This heating process is generally based on a resonance interaction of the electrons with some harmonics of the electron cyclotron waves [1]. Considerable progress has been made using this approach. There is, however, a very powerful mechanism for transferring energy and momentum from these waves to electrons, which has drawn much less attention in the past: this is the autoresonance interaction, which has been introduced by Kolomenskii and Lebedev [2], and Davydovskii [3] some decades ago.

At the origin of this mechanism is the self-synchronization process of two oscillatory motions taking place in the system: one being the relativistic gyrofrequency to which the electrons are subjected, the second is the Doppler-shifted frequency of a circularly polarized electron cyclotron wave, which the electron sees. Lately, this approach for accelerating particles has been attracting a large amount of interest [4–9]. However, the research which has been devoted to applying this autoresonance interaction to realistic plasmas is quite limited. For such plasmas, one encounters two basic obstacles: (a) the necessity of having exact appropriate initial conditions for this mechanism to be operational and (b) the necessity of having the refractive index of the medium of the propagating wave equal to 1.

In recent publications [7–9] we have shown that there is a possibility to remove the first obstacle by a mechanism, which allows for particles far away from the autoresonance (AR) conditions to get pushed in these conditions and stay there for a considerable length of time. This was obtained by introducing two circularly polarized electromagnetic waves propagating in opposite directions along a constant magnetic field. Unlike the situation encountered in the single-wave particle acceleration for which AR interaction is possible for a restricted group of particles having appropriate conditions, a two-waves configuration has a rather large flexibility for inducing the system into AR conditions. Thus, a much larger number of particles can be accelerated via AR interaction. However, in all these studies we have avoided essentially the second obstacle, namely: we have been considering a medium for which the refractive index N was put equal to 1. In the present paper, an attempt has been made to relax this restrictive condition and to apply AR interaction for realistic plasmas, concentrating on current drive schemes.

For driving a current, it is clear that one has to insure that a considerable part of the electron distribution will stream in a well-defined direction, having a rather high-averaged parallel velocity. When attempting to exploit the AR process for such a purpose, one is immediately faced with the dephasing problem, which is becoming even more severe when considering the dispersive effects. In previous publications [7–9], we have shown that for waves propagating in the vacuum, the dephasing limitation could be circumvented by introducing the special two-electron cyclotron waves configuration just mentioned. Such a configuration allows for the generation of a stochastic base, from which multiple AR accelerations can repeatedly take place and results in a velocity distribution having a high-averaged velocity parallel to the magnetic field.

Here, we will show that the same configuration itself can support a high-averaged velocity parallel to the magnetic field even for a dispersive medium. The basis for the generation of such a high stream of electrons is the existence of a trapping process taking place in the system. Indeed, when the parallel velocity of the electrons lies in the vicinity of the phase velocity of the ponderomotive propagating potential generated from the nonlinear interaction of the two waves, the electrons might get trapped in this potential and move with it. This trapping is crucial for the onset of the AR acceleration. Moreover, it turns out that under appropriate conditions, it might serve also by itself as a mechanism for generating a high-averaged velocity stream of electrons. Whether this stream of electrons will be generated by one mechanism or by the other depends on the choice of the parameters and the initial conditions of the system, as we are going to elaborate later on. Due to the essential stochastic behavior of the motion, either the multiple AR accelerations or the trapping-dominated process will be of intermittent nature.

In order to exhibit all these considerations, we present in Sec. II the underlying formalism of the analysis. Using a Hamiltonian representation of the dynamics, we derive a set of equations of motion and solve them numerically. The resulting physics uncovered by the solutions of these equations are presented either as evolving with time, or to facilitate the visualization of the trapping process, by a phase-space representation.

In order to get insight into the numerical results, we found it convenient to establish some relations concerning the dynamics of the system, which are derived in Sec. III. These relations allow us to observe in a transparent manner the importance of trapping in conditioning the system for the AR process and to distinguish between the tracks the system it is going to follow, according to the choice of parameters and initial conditions of the system. In the last section of this paper, Sec. IV, we will discuss the results and draw some conclusions on possible applications of these schemes for driving currents in dispersive thermonuclear plasmas.

II. NUMERICAL ANALYSIS

As stated in the Introduction, the configuration under study consists of two circularly polarized electromagnetic waves counterpropagating along a constant magnetic field $\vec{B}_0$, which is chosen to be in the z direction. This system has a wealth of independent parameters and initial conditions. Consequently, the uncovering of the essential physics might not be always a straightforward task. Thus, choosing an appropriate formalism might be very helpful for facilitating the analysis. We found it most convenient to use the Hamiltonian formalism to describe the dynamics of the system, since it allows us to exhibit in a natural way the relevant variables and to establish the invariants associated with this system.

The underlying physics governing the dynamics of the electrons in this configuration is essentially described by the vector potential $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} = B_0 x \vec{e_y} + A_1\{[\sin(k_1 z-\omega_1 t)]\vec{e_x} + [\cos(k_1 z-\omega_1 t)]\vec{e_y}\} + A_2\{[-\sin(k_2 z+\omega_2 t)]\vec{e_x} + [\cos(k_2 z+\omega_2 t)]\vec{e_y}\}. \quad (1)$$

In terms of this vector potential, the Hamiltonian will read

$$H = \sqrt{m^2c^4 + (c\vec{P} + e\vec{\mathbf{A}})^2}, \quad (2)$$

where A_1, A_2, ω_1, and ω_2 are, respectively, the amplitudes and the frequencies of the two propagating waves. The wave numbers k_1 and k_2 are assumed to obey the usual linear-dispersion relation associated with electron cyclotron waves,

$$k_{1,2}c = \omega_{1,2}\sqrt{1 - \frac{\omega_{p,e}^2}{\omega_{1,2}(\omega_{1,2}-\Omega)}}, \quad (3)$$

where $\Omega = eB_0/m_0c$ is the nonrelativistic gyrofrequency and $\omega_{p,e}$ is the electron plasma frequency. As the Hamiltonian does not depend explicitly on the y coordinate, its associated canonical momentum P_y is constant, which is put equal to zero without loss of generality. Since the waves are propagating in the z direction and the magnetic field is uniform, a convenient way to represent the combined gyromotion with the rotational motion of the electrons due to the electromagnetic waves is to introduce action angle variables: I and θ through the relations

$$x = \sqrt{(2cI/eB_0)}\sin(\theta), \quad P_x = \sqrt{(2eB_0 I/c)}\cos(\theta). \quad (4)$$

By eliminating time from the Hamiltonian using the generating function

$$F = (\theta + k_1 z - \omega_1 t)P_\phi + (\theta - k_2 z - \omega_2 t)P_\psi, \quad (5)$$

the transformed Hamiltonian, which is now independent of time, and thus a constant of motion, is expressed in normalized variables as

$$\overline{H}(\phi,\psi,P_\phi,P_\psi) = \sqrt{1 + \overline{P}_z^2 + (R+\rho)^2 + \sigma^2} - \overline{\omega}_1 P_\phi - \overline{\omega}_2 P_\psi, \quad (6)$$

where,

$$\phi = \frac{\partial F}{\partial P_\phi} = \theta + \overline{k}_1\overline{z} - \overline{\omega}_1\overline{t}, \quad \psi = \frac{\partial F}{\partial P_\psi} = \theta - \overline{k}_2\overline{z} - \overline{\omega}_2\overline{t}, \quad (7)$$

and the normalized actions $\overline{I}$ and $\overline{P}_z$ are expressed in terms of the momenta as

$$\overline{I} = \frac{\partial F}{\partial \theta} = P_\phi + P_\psi, \quad \overline{P}_z = \frac{\partial F}{\partial z} = \overline{k}_1 P_\phi - \overline{k}_2 P_\psi. \quad (8)$$

We have normalized the Hamiltonian $\overline{H}$ to mc^2. The normalization of the other quantities are $\overline{A}_{1,2} = eA_{1,2}/mc^2$, $\tau = \Omega t$, $\overline{\omega}_{1,2} = \omega_{1,2}/\Omega$, $\overline{k}_{1,2} = ck_{1,2}/\Omega$, $\overline{z} = \Omega z/c$, $\overline{P}_z = P_z/mc$, and $\overline{I} = I\Omega/mc^2$. We have denoted $R \equiv \sqrt{2\overline{I}}$, $\rho \equiv \overline{A}_1\sin(\phi) + \overline{A}_2\sin(\psi)$, and $\sigma \equiv \overline{A}_1\cos(\phi) + \overline{A}_2\cos(\psi)$.
The Hamilton equations of motion then read

$$\dot{\phi} = \frac{d\phi}{d\tau} = \frac{\partial\overline{H}}{\partial P_\phi} = \frac{1}{\gamma}\left(\overline{k}_1\overline{P}_z + 1 + \frac{\rho}{R}\right) - \overline{\omega}_1, \quad (9)$$

$$\dot{\psi} = \frac{d\psi}{d\tau} = \frac{\partial\overline{H}}{\partial P_\psi} = \frac{1}{\gamma}\left(-\overline{k}_2\overline{P}_z + 1 + \frac{\rho}{R}\right) - \overline{\omega}_2, \quad (10)$$

$$\dot{P}_\phi = \frac{dP_\phi}{d\tau} = -\frac{\partial\overline{H}}{\partial\phi} = -\frac{\overline{A}_1}{\gamma}[R\cos\phi - \overline{A}_2\sin(\xi)], \quad (11)$$

$$\dot{P}_\psi = \frac{dP_\psi}{d\tau} = -\frac{\partial\overline{H}}{\partial\psi} = -\frac{\overline{A}_2}{\gamma}[R\cos\psi + \overline{A}_1\sin(\xi)], \quad (12)$$

where

$$\gamma=\sqrt{[1+\bar{P}_z^2+(R+\rho)^2+\sigma^2]} \tag{13}$$

and

$$\xi=\phi-\psi. \tag{14}$$

The normalized wave numbers $\bar{k}_{1,2}$ will now be expressed as

$$\bar{k}_{1,2}=\sqrt{\bar{\omega}_{1,2}^2-\frac{e_0\bar{\omega}_{1,2}}{(\bar{\omega}_{1,2}-1)}}, \tag{15}$$

where $e_0\equiv\omega_{pe}^2/\Omega^2$. Using the constancy of the Hamiltonian $\bar{H}$, the energy of the the particle γ can be expressed as a linear combination of the actions P_ϕ and P_ψ as

$$\gamma=\bar{H}+\bar{\omega}_1P_\phi+\bar{\omega}_2P_\psi, \tag{16}$$

and the normalized parallel velocity of the particle β_z will be

$$\beta_z=\frac{\bar{P}_z}{\gamma}=\frac{\bar{k}_1P_\phi-\bar{k}_2P_\psi}{\bar{H}+\bar{\omega}_1P_\phi+\bar{\omega}_2P_\psi}. \tag{17}$$

For treating such a complex system, one naturally has to resort to numerical analyzing techniques. An effective approach for such an analysis will be to simultaneously present the evolution of the system in time and in phase space. Following this approach we have solved numerically the set of equations (9)–(12) and presented in Fig. 1(a) a portion of the time dependency of the normalized velocity β_z for a representative electron in the system. As is clearly seen, a particle having initially a relative low parallel velocity ($\beta_{z0}=0.3324$) has its velocity increased considerably attaining a time-averaged value of $\beta_z\simeq0.9$ and retaining this value for a rather long period of time. In order to understand this behavior, we present in Fig. 1(b) the corresponding evolution in time of the two actions P_ϕ and P_ψ. One readily observes here the signature of multiple AR processes. Similarly to the case where the index of refraction N is equal to 1, such an AR process is characterized by a slow variation of one of the phases (ϕ or ψ) and the fast variation of the other phase (ψ or ϕ) accordingly, thus leading to the averaged in time constancy of the corresponding action. Indeed, looking at this figure, one readily sees the constancy of the action P_ψ in the time intervals corresponding to the significant variations of the action P_ϕ. This variation is responsible for the fluctuating behavior of β_z with respect to time.

As for the condition for the system to enter into an AR process, we found it convenient to represent the dynamics of the motion in phase space. The phase-space picture β_z versus ξ (modulo 2π) is shown in Fig. 1(c). In this figure one sees that just before undertaking an AR acceleration the particle goes through a trapping process. This observation has been elaborated upon in a previous publication considering the case $N=1$. We mention here that the characteristics of such a trapping are (a) the trajectory of the particle is limited to a restricted portion of phase space and (b) the trajectory in phase space is centered around the phase velocity of the ponderomotive well, $\beta_\xi=v_\xi/c=(\bar{\omega}_1-\bar{\omega}_2)/(\bar{k}_1+\bar{k}_2)$. These characteristics hold true also for the case $N\neq1$ and will be elaborated later on.

The results just presented are not limited to a configuration having one of the frequencies larger than the gyrofrequency while the other is smaller; it holds also true when both frequencies are larger, even considerably, than the gyrofrequency. We show this in Fig. 2. In Fig. 2(a) we have presented the time dependency of the normalized parallel velocity β_z for a representative electron of the system taking $\bar{\omega}_1=3$ and $\bar{\omega}_2=2$. Comparing these results with Fig. 1(a), one realizes immediately that the generation of a high-averaged parallel velocity maintained for a long period of time is possible also in this case. As in Fig. 1, the origin of this fact can be traced back to AR interaction. This is clearly seen in Fig. 2(b) in which one sees the signature of such an interaction and the fluctuation nature of one of the actions resulting from the multiple AR process. In Fig. 2(c) we show that also here, the basis for the onset of AR acceleration is the trapping of the particle in the ponderomotive well. Let us note that for the parameters considered, β_z is rather small ($\beta_z\simeq0.2$), thus the particle entering into an AR process after getting trapped has necessarily a low value of parallel velocity. The importance of this observation will be discussed in the next section.

The underlying mechanism for the generation of a high-averaged parallel velocity retained for a rather long period of time, as presented in Figs. 1 and 2 has been the multiple AR process. In this process the trapping condition is an intermediate step for starting an AR interaction, which leads to the generation of a high parallel velocity of the particles. As we show now, it turns out that such a trapping in the ponderomotive well might be not an auxiliary step but is itself the basis of a mechanism for generating such a stream of electrons. Indeed, a trapped particle might move together with the ponderomotive propagating well, and its averaged velocity will be large if the parameters of the system are chosen such β_ξ is large.

In Fig. 3 we show that such a mechanism is operative. In frame Fig. 3(a), we readily see a similar behavior of β_z as compared to those represented in Figs. 1(a) and 2(a). One notices here that the parallel velocity is fluctuating around the phase velocity of the ponderomotive well, which is marked by a straight line in the figure. However, unlike the cases studied in Figs. 1 and 2, the time evolution of the actions P_ϕ and P_ψ are markedly dissimilar, as can be seen by inspecting Fig. 3(b). This type of time dependency is definitely of a non-AR-like characteristic. What is involved here is really a long trapping process taking place in the system as is apparent upon inspecting Fig. 3(c).

In all the examples presented in these figures either characterized by AR processes or by trapping processes, the intermittent nature of the time evolution of the system is always an inherent part of the dynamics, which reflects the possibility of the system to undergo a transition to a stochastic state.

III. THEORETICAL ASPECTS OF THE NUMERICAL RESULTS

In order to get insight into the numerical results presented in the previous section, we found it convenient to reformu-

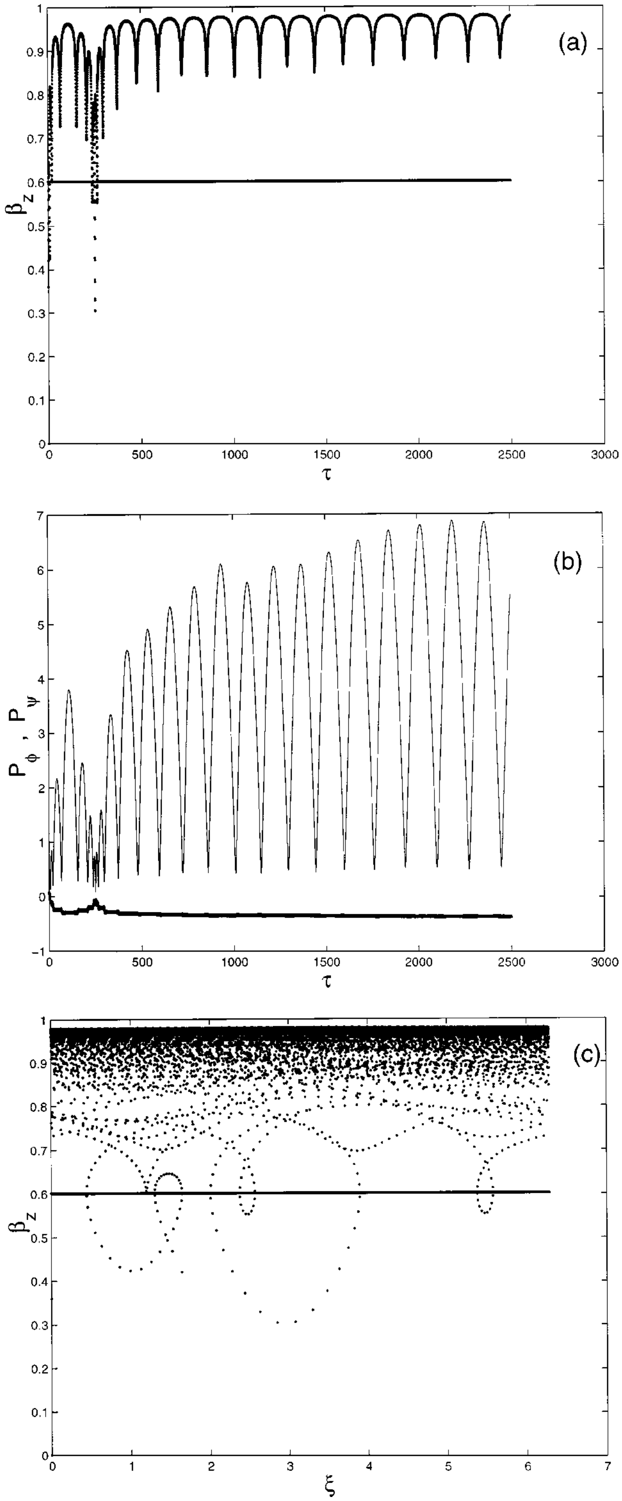

FIG. 1. (a) The change with normalized time τ of the normalized velocity in the z direction, $\beta_z = v_z/c$. The normalized parameters and initial conditions are $\overline{A}_1 = 0.45$, $\overline{A}_2 = 0.06$, $\overline{\omega}_1 = 3.0$, $\overline{\omega}_2 = 0.7$, $\phi_0 = 3.1$, $\psi_0 = 1.3$, $P_{\phi 0} = 0.2$, $P_{\psi 0} = 0.08$, and $e_0 \equiv \omega_{pe}^2/\Omega^2 = 0.1$. The wave numbers $(\overline{k_1}, \overline{k_2})$ were calculated according to Eq. (15). The straight line marks the value of the phase velocity of the ponderomotive well $\beta_\xi = v_\xi/c = (\overline{\omega}_1 - \overline{\omega}_2)/(\overline{k}_1 + \overline{k}_2)$. (b) The change with normalized time τ of the normalized actions. The thin line corresponds to the action P_ϕ and the bold line corresponds to the action P_ψ. Parameters and initial conditions as in (a). (c) The trajectory of a particle in phase space β_z versus ξ (modulo 2π). The normalized parameters and initial conditions are the same as in (a).

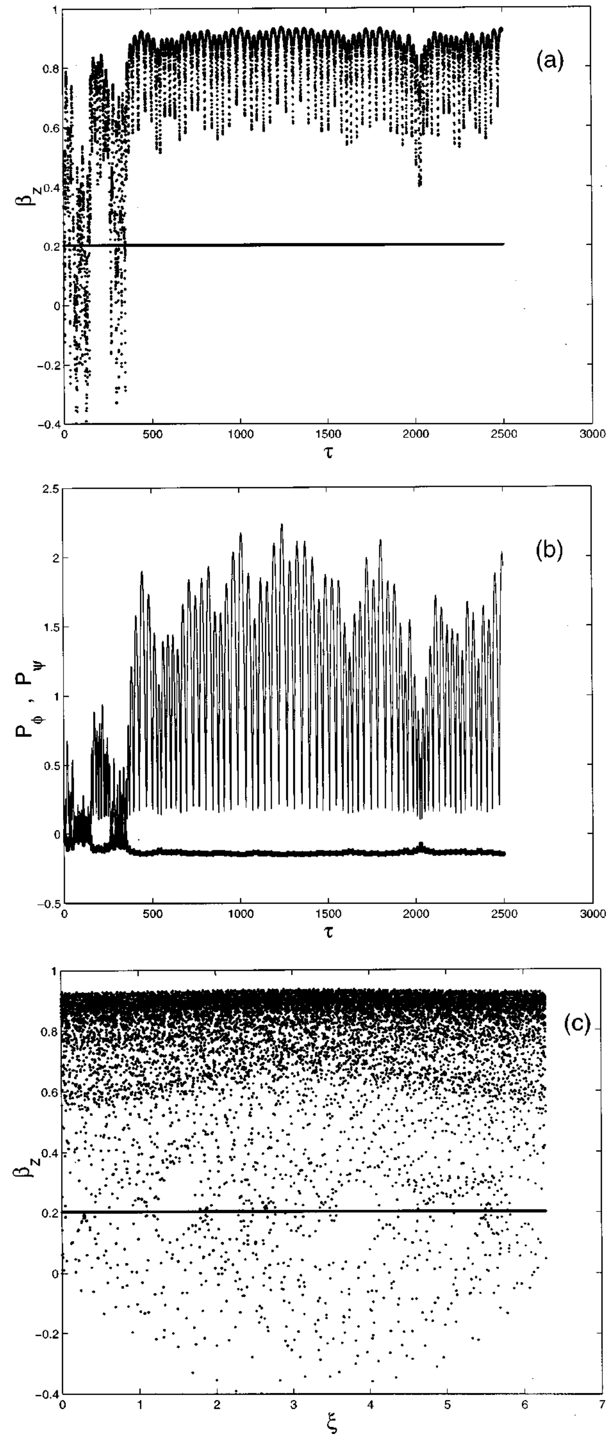

FIG. 2. (a) The change with normalized time τ of the normalized velocity in the z direction, $\beta_z = v_z/c$. The normalized parameters and initial conditions are $\overline{A}_1 = 0.45$, $\overline{A}_2 = 0.08$, $\overline{\omega}_1 = 3.0$, $\overline{\omega}_2 = 2.0$, $\phi_0 = 3.1$, $\psi_0 = 1.1$, $P_{\phi 0} = 0.03$, $P_{\psi 0} = 0.01$, and $e_0 \equiv \omega_{pe}^2/\Omega^2 = 0.09$. The wave numbers $(\overline{k_1}, \overline{k_2})$ were calculated according to Eq. (15). The straight line marks the value of the phase velocity of the ponderomotive well $\beta_\xi = v_\xi/c = (\overline{\omega}_1 - \overline{\omega}_2)/(\overline{k}_1 + \overline{k}_2)$. (b) The change with normalized time τ of the normalized actions. The thin line corresponds to the action P_ϕ and the bold line corresponds to the action P_ψ. Parameters and initial conditions are as in Fig. 2(a). (c) The trajectory of a particle in phase space β_z versus ξ (modulo 2π). The normalized parameters and initial conditions are the same as in Fig. 2(a).

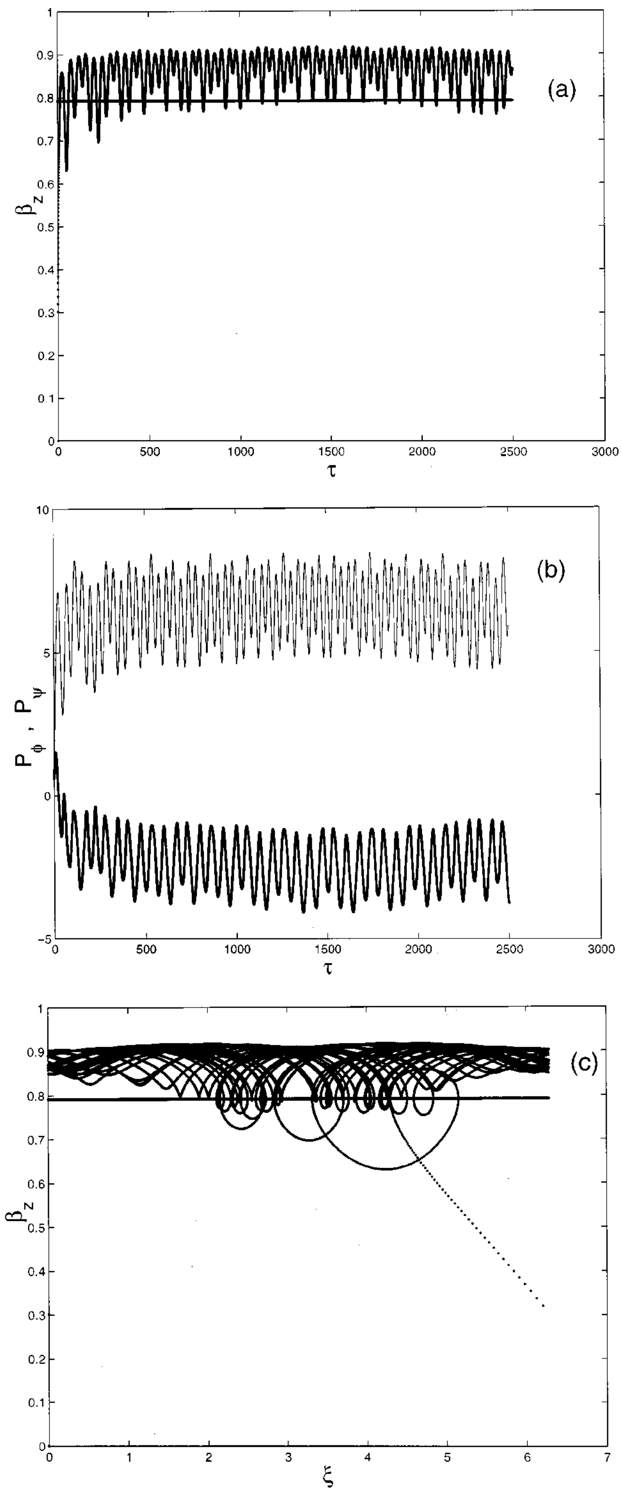

FIG. 3. (a) The change with normalized time τ of the normalized velocity in the z direction, $\beta_z = v_z/c$. The normalized parameters and initial conditions are $\bar{A}_1 = 0.5$, $\bar{A}_2 = 0.25$, $\bar{\omega}_1 = 0.7$, $\bar{\omega}_2 = 0.03$, $\phi_0 = 3.1$, $\psi_0 = 3.1$, $P_{\phi 0} = 0.9$, $P_{\psi 0} = 0.6$, and $e_0 \equiv \omega_{pe}^2/\Omega^2 = 0.06$. The wave numbers $(\bar{k}_1, \bar{k}_2)$ were calculated according to Eq. (15). The straight line marks the value of the phase velocity of the ponderomotive well $\beta_\xi = v_\xi/c = (\bar{\omega}_1 - \bar{\omega}_2)/(\bar{k}_1 + \bar{k}_2)$. (b) The change with normalized time τ of the normalized actions. The thin line corresponds to the action P_ϕ and the bold line corresponds to the action P_ψ. Parameters and initial conditions are as in Fig. 3(a). (c) The trajectory of a particle in phase space β_z versus ξ (modulo 2π). The normalized parameters and initial conditions are the same as in Fig. 3(a).

late the equations of motion, so as to make some features of the dynamics more apparent. The first step in this undertaking is to write the equations of motion in the proper time s, via the relation $ds = d\tau/\gamma$. One gets

$$\phi' = \frac{d\phi}{ds} = 1 + \frac{\rho}{R} + \bar{k}_1 \bar{P}_z - \bar{\omega}_1 \gamma, \tag{18}$$

$$\psi' = \frac{d\psi}{ds} = 1 + \frac{\rho}{R} - \bar{k}_2 \bar{P}_z - \bar{\omega}_2 \gamma, \tag{19}$$

$$P'_\phi = \frac{dP_\phi}{ds} = -\bar{A}_1 R \cos(\phi) - \bar{A}_1 \bar{A}_2 \sin(\xi), \tag{20}$$

$$P'_\psi = \frac{dP_\psi}{ds} = -\bar{A}_2 R \cos(\psi) + \bar{A}_1 \bar{A}_2 \sin(\xi). \tag{21}$$

From Eqs. (18) and (19), it follows immediately that

$$\xi' = \phi' - \psi' = (\bar{k}_1 + \bar{k}_2)(\bar{P}_z - \beta_\xi \gamma) = (\bar{k}_1 + \bar{k}_2)\gamma(\beta_z - \beta_\xi). \tag{22}$$

The relevant physical momenta variables of the system are the momentum in the perpendicular direction $\bar{I}$ and the parallel momentum $\bar{P}_z$. However, from the numerical analysis, the importance of the trapping process was revealed and, therefore, the variable expressing the parallel electron velocity relative to the phase velocity of the ponderomotive well plays a crucial role. As can be seen from Eq. (22), ξ' is a measure of this relative velocity. The preferred variables of the analysis will thus be $\bar{I}$ and ξ'. From Eqs. (8) and (13) we express γ as

$$\gamma = \bar{H} + \lambda \bar{I} + \beta_\xi \bar{P}_z, \tag{23}$$

where $\lambda \equiv (\bar{k}_1 \bar{\omega}_2 + \bar{k}_2 \bar{\omega}_1)/(\bar{k}_1 + \bar{k}_2)$. Using now Eqs. (22) and (23), we express $\bar{P}_z$ and γ in terms of ξ' and $\bar{I}$ as follows:

$$\bar{P}_z = \frac{\xi'}{(\bar{k}_1 + \bar{k}_2)(1 - \beta_\xi^2)} + \frac{\beta_\xi(\bar{H} + \lambda \bar{I})}{(1 - \beta_\xi^2)}, \tag{24}$$

$$\gamma = \frac{\beta_\xi \xi'}{(\bar{k}_1 + \bar{k}_2)(1 - \beta_\xi^2)} + \frac{(\bar{H} + \lambda \bar{I})}{(1 - \beta_\xi^2)}. \tag{25}$$

With the help of these expressions, the equations of motion Eqs. (18) and (19) get the form

$$\phi' - \frac{\rho}{R} = \frac{\bar{I}_s - \bar{I}}{\mu} + \frac{u_1}{u_1 + u_2}\xi', \tag{26}$$

$$\psi' - \frac{\rho}{R} = \frac{\bar{I}_s - \bar{I}}{\mu} - \frac{u_2}{u_1 + u_2}\xi', \tag{27}$$

where $\mu \equiv (1 - \beta_\xi^2)/\lambda^2$, $u_1 \equiv (\bar{k}_1 - \bar{\omega}_1 \beta_\xi)/\lambda$, $u_2 \equiv (\bar{k}_2 + \bar{\omega}_2 \beta_\xi)/\lambda$, and $\bar{I}_s \equiv \mu - \bar{H}/\lambda$. Inserting the expression of $\bar{P}_z$ and γ as given by Eqs. (24) and (25) in Eq. (22), one gets

$$\xi'^2=(\overline{k}_1\overline{\omega}_2+\overline{k}_2\overline{\omega}_1)^2\{(\overline{I}-\overline{I}_s)^2+c_0 -2\mu[R\rho+\overline{A}_1\overline{A}_2\cos(\xi)]\}, \tag{28}$$

where $c_0\equiv\mu(2\overline{H}/\lambda-\mu-1-\overline{A}_1^2-\overline{A}_2^2)$.

From Eq. (18) one sees straightforwardly that the normalized AR condition $\gamma(1-N_1\beta_z)-1/\overline{\omega}_1\simeq 0$ implies $\phi'-\rho/R\simeq 0$, and similarly, from Eq. (19), $\gamma(1+N_2\beta_z)-1/\overline{\omega}_2\simeq 0$ implies $\psi'-\rho/R\simeq 0$.

Inspecting now the expression for the AR factor $\phi'-\rho/R$ as given in Eq. (26), one observes that this expression can be separated in two groups: one includes a contribution in the parallel direction $u_1\xi'/(u_1+u_2)=u_1\lambda(\overline{P}_z-P_\xi)/\mu$, where $P_\xi=\gamma\beta_\xi$, and the other includes a contribution in the perpendicular direction: $(\overline{I}_s-\overline{I})/\mu$. A similar structure is seen in Eq. (27). From Eqs. (26) and (27) one sees that having a very small value of $u_{1,2}\xi'/(u_1+u_2)$ is by itself not sufficient to insure the onset of an AR interaction (although it is essential for conditioning the system, via trapping, for such an interaction). The contribution, which includes perpendicular terms $(\overline{I}_s-\overline{I})/\mu$, is an important factor in determining whether the system will go through an AR or not. Only when the combination $(\overline{I}_s-\overline{I})/\mu\pm u_{1,2}\xi'/(u_1+u_2)$ is small can one expect the start of an AR process. Once the particle has been conditioned by trapping and gone through an AR interaction, the velocity fluctuations, which the particle undergoes afterwards, does not following a trapping process. This is clearly seen in Figs. 1(c) and 2(c). However, as these fluctuations might be consequential to the quality of the current generated from the application of such an acceleration mechanism, it is important to get an estimate of the fluctuating levels of the action and thus on the velocity attained by the particle. During an AR process, one of the frequencies, say, ϕ' is slowly varying while ψ' is undergoing fast oscillations. Averaging now the equation for the action P_ψ [Eq. (21)] with respect to these fast oscillations, reveals the existence of an additional adiabatic invariant $P_\psi\simeq P_{\psi,c}$, where $P_{\psi,c}$ is the averaged constant value of the action P_ψ during the AR process. This constant together with the exact invariant $\overline{H}$ makes the system integrable, thus allowing for estimating the variation of the action P_ϕ with time. Explicitly, after averaging the equations with respect to the fastly varying variable (ψ), one ends up with the unknown variables P_ϕ and ϕ. It is convenient, however, to perform the analysis by using $\overline{I}=P_\phi+P_\psi$ and ϕ. A relation between these two variables is obtained by averaging the expression for γ^2.

$$\gamma^2=(\overline{H}+\overline{\omega}_1P_\phi+\overline{\omega}_2P_{\psi,c})^2=1+(\overline{k}_1P_\phi-\overline{k}_2P_{\psi,c})^2 +(R+\rho)^2+\sigma^2; \tag{29}$$

one then gets

$$(\overline{k}_1^2-\overline{\omega}_1^2)\frac{\overline{I}^2}{2}+\overline{I}d_0+\overline{A}R\sin(\phi)=d_1, \tag{30}$$

where $d_0\equiv 1-\overline{\omega}_1\overline{H}-\lambda u_1(\overline{k}_1+\overline{k}_2)P_{\psi,c}$ and

$$d_1\equiv\{[\overline{H}+(\overline{\omega}_2-\overline{\omega}_1)P_{\psi,c}]^2-1\text{-}\overline{A}_1^2-\overline{A}_2^2-(\overline{k}_1+\overline{k}_2)^2P_{\psi,c}^2\}/2.$$

Using then the evolution equation for $\overline{I}$,

$$\overline{I'}=-R[\overline{A}_1\cos(\phi)+\overline{A}_2\cos(\psi)], \tag{31}$$

one finds, after averaging,

$$\overline{I'}\simeq-\overline{A}_1R\cos(\phi). \tag{32}$$

Eliminating ϕ between Eqs. (30) and (32) one ends up with the following differential equation:

$$\overline{I'}^2=-(\overline{\omega}_1^2-\overline{k}_1^2)^2\frac{\overline{I}^4}{4}+d_0(\overline{\omega}_1^2-\overline{k}_1^2)\overline{I}^3 -[d_0^2+d_1(\overline{\omega}_1^2-\overline{k}_1^2)]\overline{I}^2+2(d_0d_1+\overline{A}_1^2)\overline{I}-d_0^2. \tag{33}$$

This equation can be solved in terms of Jacobi elliptic functions, but for our purpose for estimating the fluctuations of the parallel velocity, or in other words the extreme values of P_ϕ, it is sufficient to find the roots of the four degrees polynomial equation $\overline{I'}^2=0$. Considering for example an AR structure of P_ϕ as a function of time, corresponding to the parameters of Figs. 1 and using an averaged value $P_{\psi,c}=-0.357$, the real roots of Eq. (33) fix the maximum value of P_ϕ to be 6.5232 and its minimum value to be 0.4853. These results are in good agreement with the values found in Fig. 1(b), thus verifying the predictive possibility of this formalism, to estimate the fluctuations levels of the velocity of the accelerated particle.

For the generation of streams of electrons having high velocities in the parallel direction via an AR interaction, the conditions insuring the smallness of $\phi'-\rho/R$ or $\psi'-\rho/R$ have to be fulfilled. However, not always is it necessary to have $\phi'-\rho/R$ or $\psi'-\rho/R$ small in order that an electron will be able to acquire a high-averaged parallel velocity and to retain it. Under appropriate conditions the trapping of the electron in the ponderomotive well, namely, when $\xi'\simeq 0$, by itself leads to the same result. Indeed, we have presented in Fig. 3 a case where the trapping of the electron in the ponderomotive well occurs during large portions of time, without inducing any AR process. For having a long period trapping, the condition $\xi'\simeq 0$ by itself is not sufficient. A second condition, which might insure this long trapping, is that $\overline{I}=P_\phi+P_\psi\simeq$Const in average. This condition results from the absence of any resonating interaction in the system. Thus, the two frequencies ϕ' and ψ' are necessarily quite high, which implies the constancy of the averaged action $\overline{I}$, as can be seen by averaging Eq. (31). Under these conditions, and averaging with respect to the fast oscillations ϕ and ψ, Eq. (28) will then take the form of a pendulum equation:

$$\xi'^2=(\overline{k}_1\overline{\omega}_2+\overline{k}_2\overline{\omega}_1)^2[(\overline{I}_t-\overline{I}_s)^2+c_0-2\mu\overline{A}_1\overline{A}_2\cos(\xi)], \tag{34}$$

where we have approximately set $\overline{I}=\overline{I}_t$, the value of the averaged action at the bottom of the ponderomotive well.

These considerations can be validitated in a transparent manner in the example presented in Fig. 3. Indeed, one sees there the fluctuations of the two actions P_ϕ and P_ψ around an averaged constant value, which we denote as $P_{\phi,t}$ and $P_{\psi,t}$. These values can be derived from the underlying assumptions just presented, namely, [$\xi'=0$; $\overline{I}=\overline{I_t}$]. These two conditions amount really to a set of two linear equations in P_ϕ and P_ψ. The solution of this set of equations is straightforwardly given by

$$P_{\phi,t}=\frac{u_2\lambda\overline{I_t}+\beta_\xi\overline{H}}{\lambda(u_1+u_2)}, \tag{35}$$

$$P_{\psi,t}=\frac{u_1\lambda\overline{I_t}-\beta_\xi\overline{H}}{\lambda(u_1+u_2)}. \tag{36}$$

The value of $\overline{I_t}$ can be estimated with the help of Eq. (34). This equation is best satisfied when $\xi'=0$, since in this case, setting $\overline{I}=\overline{I_t}$ in the derivation of the equation is an exact operation, while it is an approximate one when $\xi'\neq 0$. Setting $\xi'=0$ in Eq. (34) one gets

$$\overline{I_t}=\overline{I_s}\pm\sqrt{-c_0+2\mu\overline{A_1}\overline{A_2}\cos(\xi)}.$$

For the parameters of Fig. 3, $c_0 \gg 2\mu\overline{A_1}\overline{A_2}$; hence, one gets $\overline{I_t}\simeq\overline{I_s}-\sqrt{-c_0}$. Inserting the parameters and initial conditions of Fig. 3 in expressions (35) and (36), we find $P_{\phi,t}=5.103$ and $P_{\psi,t}=-0.778$, in agreement with the averaged values of P_ϕ and P_ψ, as can be seen in Fig. 3(b). Let us notice that, at this interval of time, the trapping of the electrons takes place indeed, as can be seen in Fig. 3(c).

IV. DISCUSSION

In this study, we have shown the possibility of generating a high-averaged parallel velocity stream of electrons for long periods of time. The configuration investigated consists of two circularly polarized electromagnetic waves counterpropagating along a constant magnetic field in a dispersive medium. Two different mechanisms have been found to be responsible for this generation of streams: one having its origin in the AR acceleration of electrons that follows a conditioning trapping stage, the second being dominated by the trapping process itself.

The trapping process takes place in the ponderomotive well generated by the nonlinear interaction of the two waves. The velocity and directionality of this propagating well is determined by the parameters of the system, which are under the control by the experimentalists. Choosing properly these parameters, one can ascertain a motion in a given direction, thus generating a preferential motion along the constant magnetic field in the system. Such a preferential motion is, of course, essential for driving an effective current. When considering the trapping mechanism of generating streams, the choice of the frequencies, which fixes a value of β_ξ of the ponderomotive well, is determining by itself the effectiveness and direction of the current. While for the AR mechanism, the amplitudes of the waves affect also these characteristics. Indeed, having made a choice of the relative magnitudes of $\overline{\omega}_1$ and $\overline{\omega}_2$, the corresponding choice of the amplitudes $\overline{A}_1$ and $\overline{A}_2$ can insure a more effective current drive, by inhibiting AR accelerations in an unwanted direction. From these considerations, the usefulness of reaching a high average β_z is quite clear. We have shown in Fig. 1(a) that such values of β_z are readily attainable.

Comparing Figs. 1(a) and 1(b) one might at first be somewhat surprised by the high value of the averaged parallel velocity of the electron as seen in Fig. 1(a) although the values of the action P_ϕ as seen in Fig. 1(b) are quite low. This result is self-explanatory when considering Eq. (17) where one realizes that for the case considered, P_ψ is negative and thus effective in making β_z relatively high.

It is worth mentioning that the high β_ξ value is not always necessary for inducing a high-averaged β_z, as can be seen in Fig. 2. Here, one has the advantage of accelerating via multiple AR process, particles with low initial velocities of which are numerous in any system to be considered. Let us note that such values of parameters as used in the case considered in Fig. 2 might be relevant for an experimental setup having two gyrotrons transmitting two electromagnetic waves, one in the range of 118 GHz and the other in the range of 82.7 GHz. Electromagnetic waves with such frequencies are currently applied in the tokamak a configuration variable (TCV) experiment in Lausanne (Switzerland) [10]. Some aspects of the advantages of using two counterpropagating electromagnetic waves have been elaborated upon in previous publications [7–9]. Here, we mention especially the possibility of increasing considerably the particle population, which might be accelerated by a two-waves interaction process, as compared to the single-wave case. Moreover, we show here that the two-waves configuration allows us to circumvent the deterioration of the acceleration of the electrons, which is characteristic for a dispersive medium. This is achieved by making two mechanisms operational each permitting a rather high parallel velocity built up with a not too large acceleration, utilizing a trapping process of the particles in the ponderomotive well generated from the nonlinear interaction of the two waves. The velocity so acquired might not be enough for accelerators but might be quite sufficient for driving currents.

Let us mention that associated with the two-waves interaction is an inherently built-in potential for a stochastic behavior. For both mechanisms considered in this paper either the AR interaction or the trapping process, this stochastic behavior is reflected in the intermittent nature of these mechanisms. This fact diminishes considerably, in the framework of a single-particle analysis, the predictability one has with regard to the initial conditions of the electron, which will allow us then to contribute effectively to the generation of a current. Indeed, electrons that have initial conditions, which would not allow them to be trapped in the ponderomotive well as described by Eq. (34), might wander in the stochastic sea and arrive at privileged conditions, which will allow them to get trapped and to contribute to the generation of the current.

Moreover, even electrons, which participate already in the current drive via the autoresonance or intermittent trapping mechanisms, might, due to the stochastic behavior of the interaction, not contribute effectively, for short periods of time to the current in the plasma. This is clearly seen in Fig. 1(a). However, when a distribution of many electrons is considered, at a time when one particle is not contributing effectively to the current, many other particles having slightly different initial conditions will compensate for the deficiency of this particle contribution allowing for the generation of a well-behaved current. Thus, the question regarding the conditions, which the particles have to fulfill in order to contribute effectively to the generation of a current, has to be addressed, not in the framework of a single-particle analysis that describes that basic effects but in studies considering a distribution of particles, where averaging procedures can lead to reliable estimates of these conditions. Such a study is presently undertaken. Here, we would like only to mention that preliminary numerical results indicate that, while not being a desired feature of the interaction, the real impact of the intermittence of the motion of the electrons on the effectiveness of generating a current is very limited when considering a large distribution of electrons followed for a long period of time.

The results of the work presented here are quite encouraging, however; the applicability of this scheme demands further studies considering realistic plasmas. Here, one faces the basic problem of launching two waves with different frequencies and wave numbers into a toroidal device. Fortunately, with regard to this problem we can benefit from the research done and experience gained in the beat-wave current drive project advanced by research teams of the Lawrence Livermore National Laboratory [11]. In the beat-waves current generation scheme, two intense electromagnetic waves interact nonlinearly in the plasma. When these electromagnetic waves have a difference in frequencies that is approximately equal to the electron plasma frequency, low-frequency longitudinal waves may be resonantly driven. If the phase velocity is properly chosen, these waves may damp and accelerate electrons, thus driving a current.

Although completely different from our scheme of driving a current in which the acceleration of the electrons is done by the electromagnetic waves themselves, the technique of launching the waves in the plasma might be similar. The schemes for launching the waves as presented by Thomassen for the Microwave Tokamak Experiment [12] and the one realized in the experiment performed on the Davis Diverted Torus [13] may be very useful for planning and constructing and experimental setup, which will enable us to test the scheme for driving currents as proposed in this paper.

Detailed studies of the interaction of two electromagnetic waves launched in a plasma having a hot relativistic distribution of electrons are presently under way.

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